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THE RUNTS OF THE LITTER: WHY PLANETS FORMED THROUGH GRAVITATIONAL INSTABILITY CAN ONLY BE FAILED BINARY STARS

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ABSTRACT

Recent direct imaging discoveries suggest a new class of massive, distant planets around A stars. These widely separated giants have been interpreted as signs of planet formation driven by gravitational instability, but the viability of this mechanism is not clear cut. In this paper, we first discuss the local requirements for fragmentation and the initial fragment mass scales. We then consider whether the fragment's subsequent growth can be terminated within the planetary mass regime. Finally, we place disks in the larger context of star formation and disk evolution models. We find that in order for gravitational instability to produce planets, disks must be atypically cold in order to reduce the initial fragment mass. In addition, fragmentation must occur during a narrow window of disk evolution, after infall has mostly ceased, but while the disk is still sufficiently massive to undergo gravitational instability. Under more typical conditions, disk-born objects will likely grow well above the deuterium burning planetary mass limit. We conclude that if planets are formed by gravitational instability, they must be the low mass tail of the distribution of disk-born companions. To validate this theory, on-going direct imaging surveys must find a greater abundance of brown dwarf and M-star companions to A-stars. Their absence would suggest planet formation by a different mechanism such as core accretion, which is consistent with the debris disks detected in these systems.

Subject headings: planet formation, accretion disks, binaries

1. INTRODUCTION

Motivated by the recent discovery of massive planets on wide orbits, we explore the requirements for making gas giants at large separations from their host star via gravitational instability, hereafter, GI. In particular, we consider the formation mechanism for the system HR 8799 which contains three $\sim 10 M_{\rm Jup}$ objects orbiting at distances between ~ 30 and 70 AU (Marois et al. 2008). The standard core accretion model for planet formation, already strained in the outer solar system, has difficulty explaining the presence of these objects. While GI is an unlikely formation mechanism for close in planets (Rafikov 2005), for more widely separated planets, or sub-stellar companions, the viability GI-driven fragmentation deserves further investigation.

In the inner regions of a protoplanetary disk, gas cannot cool quickly enough to allow a gravitationally unstable disk to fragment into planets (Rafikov 2005; Matzner & Levin 2005). For this reason, core accretion—in which solid planetesimals collide and grow into a massive core which then accretes a gaseous envelope—has emerged as the preferred mechanism for forming planets at stellar separations $\lesssim 10$ AU. Planets at wider separations have only recently been discovered by direct imaging around the A-stars HR 8799 (Marois et al. 2008), Fomalhaut (Kalas et al. 2008), and possibly Beta Pic (Lagrange et al. 2009). Searches at large radii surrounding solar-type stars have yet to turn up similar companions (Nielsen & Close 2009). Standard core accretion models cannot form these planets, though further investigation is warranted.

In favor of this possibility, all three systems show some evidence of processes related to core accretion: all have infrared excess due to massive debris disks at large radii. This is at least partially a selection effect as these systems were targeted due to the disks' presence. Nevertheless, these debris disks are composed of reprocessed grains from collisions of planetesimals. The disks' long lifetimes prohibit a primordial origin for small grains—they are removed quickly by radiation pressure and Poynting-Roberston drag (Aumann et al. 1984) and so must be regenerated from collisions between larger bodies that formed through the coagulation of solids at early times. Therefore, planetesimal formation, a necessary ingredient in core accretion models, has taken place (e.g., Youdin & Shu 2002; Chiang & Youdin 2009).

In addition, other A-stars host planets at 1–2 AU (Johnson et al. 2007) which, although they have a distinct semi-major axis distribution from planets orbiting G and M stars, are likely formed by core accretion. If future surveys demonstrate that this distribution extends smoothly to wide separation planets, then simplicity would argue against a distinct formation mechanism for the wide giants.

Yet the standard core accretion model faces a serious

problem at large distances. The observed lifetimes of gas disks are short, at most a few Myr (Hillenbrand et al. 1992; Jayawardhana et al. 2006). In contrast, typical core accretion times increase with radius and exceed 10 Myr beyond 20 AU (e.g. Levison & Stewart 2001; Goldreich et al. 2004). Whether or not this theoretical difficulty can be overcome will require careful modeling of the interactions between planetesimals and the young gas disk.

Could wide orbit planets have formed at smaller radii, and migrated outwards? Dodson-Robinson et al. (2009), have investigated the possibility of forming the HR 8799 system via scattering in the absence of dynamically important gas, but find that putting three massive planets into such closely spaced yet wide orbits is unlikely. Crida et al. (2009) have suggested that under favorable circumstances outward migration in resonances might be feasible. Alternatively, the core of a giant planet could be scattered outward by a planet, or migrate outward before accreting its gas envelope, either by interactions with the gas disk (Type III migration; e.g. Masset & Papaloizou 2003) or with planetesimals embedded in the gas (Capobianco, Duncan, & Levison, in prep). Neither mechanism has yet been shown to move a core to such large distances, though this possibility has not been ruled out.

Given these difficulties, it is natural to search for other formation mechanisms, and GI (Boss 1997) stands out as a promising alternative. If any planets form by GI, the recently discovered directly imaged planets are the most likely candidates (Rafikov 2009; Boley 2009; Nero & Bjorkman 2009). In this paper, we examine this possibility in more detail, considering the expected mass scale of fragments and the effect of global disk evolution on the formation process.

The inferred masses for the HR 8799 planets are close to the deuterium burning limit of $13M_{\rm Jup}$ (Chabrier & Baraffe 2000). For simplicity, we take this as a the dividing line between planets and brown dwarfs and we refer to the HR 8799 objects as planets throughout. However, there is no reason for a given formation mechanism to function only above or below this threshold, and in fact, we will argue that if the HR 8799 planets formed by GI, their histories are more akin to those of higher-mass brown dwarfs than to lower-mass planets.

To constrain GI as a mechanism for wide giant planet formation, we set the stage by describing the HR 8799 system in §2. We review the standard requirements for fragmentation in §3 and discuss the initial mass scale of fragments in §4. In §5 we show that under typical disk conditions, fragments will continue to accrete to higher masses. We then discuss important global constraints on planet formation provided by star formation models, disk evolution timescales, and migration mechanisms in §6, and §7. We compare predictions of our analysis with the known wide substellar companions and exoplanets in §8, suggesting that future observations will provide a definitive answer to the formation mechanism for HR 8799. In the appendices we re-examine the heating and cooling properties of disks that are passively and actively heated, with special attention to the implications for irradiated disks, which become increasingly relevant for more massive stars.

2. THE HR 8799 SYSTEM

The planets around HR 8799 probe a previously unexplored region of parameter space (Marois et al. 2008; Lafrenière et al. 2009) because they are more distant from their host star. The three companions to HR 8799 are observed at separations from their host star of 24, 38, and 68 AU. Their masses, estimated using the observed luminosities of the planets in conjunction with cooling models, have nominal values of 10, 10, and 7 $M_{\rm Jup}$, respectively. A range in total mass of 19–37 $M_{\rm Jup}$ is derived from uncertainties in the age of the host star (Marois et al. 2008). Interpretation of the cooling models generates substantial additional systematic uncertainty—recent measurements suggest that these models may overpredict the masses of brown dwarfs by $\sim$25% (Dupuy et al. 2009). Fabrycky & Murray-Clay (2008) have demonstrated that for planetary masses in the stated range, orbital stability over the age of the system requires that the planets occupy at least one mean-motion resonance, and that for doubly-resonant orbital configurations total masses of up to at least $54M_{\rm Jup}$ can be stable.

HR 8799 has been called a "scaled-up solar system" in terms of the stellar flux incident on its giant planets (Marois et al. 2008; Lafrenière et al. 2009). However for understanding the formation of this system it is more useful to consider dynamical times, and disk mass requirements. Because the dynamical time at fixed radius scales only as $M_*^{1/2}$, the dynamical times are larger at the locations of the HR 8799 planets that at the solar system giants. Since the total mass in planets greatly exceeds the $\sim 1.5\ M_{\rm Jup}$ in the solar system, we can infer that (as with some other extrasolar systems) the primordial disk around HR 8799 was more massive than the solar nebula and/or there was greater efficiency of planet formation, especially in the retention of gas. Compared to solar system giants, longer dynamical times make core accretion more difficult and larger disk masses make GI more plausible.

3. IDEAL CONDITIONS FOR GI-DRIVEN FRAGMENT FORMATION

We first determine where, and under what local disk conditions, fragmentation by GI is possible. Following Gammie (2001), Matzner & Levin (2005) and Rafikov (2005), we argue that for a disk with surface density Σ and temperature T to fragment, it must satisfy two criteria. First, it must have enough self-gravity to counteract the stabilizing forces of gas pressure and rotational shear, as quantified by Toomre's Q:

$$Q \equiv \frac{c_s \Omega}{\pi G \Sigma} < Q_o \sim 1 \quad (1)$$

(Safronov 1960; Toomre 1964), where $c_s = \sqrt{kT/\mu}$ is the isothermal sound speed of the gas with mean particle weight $\mu = 2.3 m_{\rm H}$ appropriate for a molecular gas, G is the gravitational constant, k is the Boltzmann constant, and Ω is the orbital frequency. Equation (1) specifies the onset of axisymmetric instabilities in linear theory that can give rise to bound clumps (Goldreich & Lynden-Bell 1965).[1]

[1] In a realistic disk model, clumps likely form within spiral arms formed via non-axisymmetric, non-linear instabilities, although the

The second criterion that must be satisfied for fragmentation to proceed is the so-called cooling time criterion. The heat generated by the release of gravitational binding energy during the contraction of the fragment must be radiated away on the orbital timescale so that increased gas pressure does not stall further collapse (Gammie 2001). This implies

$$t_{\rm cool} = \frac{3\gamma\Sigma c_s^2}{32(\gamma-1)}\frac{f(\tau)}{\sigma T^4} \lesssim \zeta\Omega^{-1}. \quad (2)$$

Here ζ is a constant of order unity, γ is the adiabatic index of the gas, and σ is the Stefan-Boltzmann constant. We take $f(\tau) = 1/\tau + \tau$ (Rafikov 2005) for disk vertical optical depth $\tau = \kappa\Sigma/2$ and gas opacity κ. Numerical models of collapse in barotropic disks measure the critical value ζ through the inclusion of a loss term $u/t_{\rm cool}$ in the equation for the internal energy, u. Estimates of ζ range from $\sim 3-12$, depending on γ (Rice et al. 2005), the numerical implementation of cooling (Clarke et al. 2007), and the vertical stratification in the disk. We assume $\gamma = 7/5$, appropriate for molecular hydrogen, and we adopt $\zeta = 3$ here. Although ζ was measured in disks whose temperature is controlled by viscous heating, we show in Appendix A that the same expression (modulo slightly different coefficients) should apply when irradiation sets the disk temperature, as will be the case in disks prone to fragmentation (see Appendix B).

A disk satisfying Toomre's criterion for instability (equation 1) but not the cooling time criterion (equation 2) experiences GI-driven angular momentum transport which regulates the surface density of the disk so that $Q \sim Q_o \sim 1$ and Q does not reach substantially smaller values (c.f. Appendix A). We can therefore use Toomre's criterion to define a relationship between Σ and T at fragmentation, as a function of period:

$$\Sigma = \frac{c_s\Omega}{\pi G Q_o} = f_q\sqrt{T}\Omega \quad (3)$$

where for convenience we define $f_q \equiv (k/\mu)^{1/2}(\pi G Q_o)^{-1}$. We shall hereafter set $Q_o = 1$.

Given equation (3), we can rewrite equation (2) to generate a single criterion for fragmentation which depends on temperature and location:

$$\frac{\Omega t_{\rm cool}}{\zeta} = (f_q f_t)\ \frac{\Omega^2}{T^{5/2}} f(\tau) \leq 1, \quad (4)$$

where $f_t \equiv (3/32)\gamma(\gamma-1)^{-1}k(\mu\sigma\zeta)^{-1}$. Somewhat counterintuitively, the critical cooling constraint requires that a disk be sufficiently hot to fragment. The value of $f(\tau)$ depends on both Ω and T. Evaluating this criterion relies on the disk opacity, which we return to in §4.2.

4. MINIMUM FRAGMENT MASSES AND SEPARATIONS

4.1. *Initial masses of GI-born fragments*

We take the initial mass of a fragment to be the mass enclosed within the radius of the most unstable wavelength, $\lambda_Q = 2\pi H$ in a $Q = 1$ disk, or:

$$M_{\rm frag} \approx \varSigma(2\pi H)^2 \quad (5)$$

(Levin 2007), where $H = c_s/\Omega$ is the disk scaleheight. Cossins et al. (2009) have shown that even when the GI is non-axisymmetric, the most unstable axisymmetric wavelength λ_Q is one of the dominant growing modes, suggesting that this is a reasonable estimate. While more numerical follow up will be necessary to pin down the true distribution of fragments born through GI, at present simulations show that this estimate may well be a lower limit, but is the correct order of magnitude (Boley 2009; Stamatellos & Whitworth 2009).

Using equation (3), we rewrite the fragment mass explicitly as a function of temperature and location:

$$M_{\rm frag} \approx 4\pi\left(\frac{H}{r}\right)^3 M_* = f_m\frac{T^{3/2}}{\Omega} \quad (6)$$

where $f_m \equiv (2\pi)^2 f_q k/\mu$. Equation (6) demonstrates that at a given disk location, fragment masses depend only on temperature, with lower temperatures generating smaller fragments, subject to the minimum temperature required for fragmentation by equation (4).

Rafikov (2005) pointed out that there exists an absolute minimum fragment mass at any disk location, when the disk satisfies the equalities in equation (1) and equation (2) and has $\tau = 1$. The minimum temperature required for fragmentation scales as $T \propto (\tau+1/\tau)^{2/5}$ (equation 4), so the critical temperature and fragment mass are minimized at $\tau = 1$, the optical depth for which cooling is most efficient. The corresponding minimum mass as a function of location is:

$$M_{\rm f,min} = f_m(f_q f_t)^{3/5}\Omega^{1/5} \quad (7)$$

$$= 1.5 M_{\rm Jup}\left(\frac{r}{100\ \rm AU}\right)^{-3/10}\left(\frac{M_*}{1.5 M_\odot}\right)^{1/10} \quad (8)$$

which occurs for disk temperatures:

$$T_{\rm f,min} = 7{\rm K}\left(\frac{\rm r}{100\ \rm AU}\right)^{-6/5}\left(\frac{\rm M_*}{1.5 M_\odot}\right)^{2/5} \quad (9)$$

Equation (7) corresponds to $Q = 1$, $\Omega t_{\rm cool} = 3$, and $\tau = 1$. This minimum mass is only achieved for temperatures consistent with $T_{\rm f,min}$. Once an opacity law is specified which relates T and τ, the problem becomes overconstrained— these three criteria can only be satisfied at a single disk location, and equations 7-9 are valid at only one radius in the disk. We now proceed to evaluate the critical disk temperatures and fragment masses for realistic opacity laws, demonstrating that planetary-mass fragments can only form at large separations from their host star.

4.2. *Opacity*

At low temperatures, when $T \lesssim 155$ K, ice grains are the dominant source of opacity (Pollack et al. 1985). Above $\sim$155 K, ices begin to sublimate. In the cold regime applicable to the outer regions of protoplanetary disks, we consider two realistic opacity laws, one which is characteristic of grains in the interstellar medium (ISM), and one which is characteristic of grains that have grown to larger sizes due to processes within the disk. We assume a Rosseland mean opacity which scales as

$$\kappa \approx \kappa_\beta T^\beta \quad (10)$$

critical value of Q at which fragmentation occurs should remain similar.

where the both the exponent, β, and the constant κ_β are not well constrained in protoplanetary disks. They depend on the number-size distribution and composition of the dust grains, and on the dust-to-gas ratio. For ISM like grains, the Rosseland mean opacity may be approximated by an opacity law with $\beta = 2$, or

$$\kappa \approx \kappa_2 T^2 \tag{11}$$

(Pollack et al. 1996; Bell & Lin 1994; Semenov et al. 2003) as long as the ice grains dominating the opacity are smaller than a few tens of microns. This opacity law is observationally confirmed in the ISM (Beckwith et al. 2000, and references therein). For our fiducial model we use $\kappa_2 \approx 5\times10^{-4}\mathrm{cm}^2/\mathrm{g}$ for T in Kelvin, a fit to the standard dust model by Semenov et al. (2003). Throughout, we quote κ per gram of gas for a dust-to-gas ratio of 10^{-2}.

As grains grow larger, they eventually exceed the wavelength of the incident radiation, and so the opacity is determined by the geometric optics limit. In this case the Rosseland mean opacity is independent of temperature, so the exponent $\beta = 0$. In this limit:

$$\kappa \approx \kappa_0 \tag{12}$$

For a fixed dust mass in grains of size s, $\kappa_0 \propto s^{-1}$. For our fiducial model we choose $\kappa_0 \approx 0.24$ cm^2/g which is valid for $T \gtrsim 20$ K and typical grain sizes of order $300\mu m$ (see Figure 6 of Pollack et al. 1985).

Observations of emission from optically thin protoplanetary disks show evidence of grain growth at millimeter wavelengths. Specifically the measured opacities $\kappa_\nu \propto \nu^\alpha$ with $\alpha \simeq 0.5$—1.5 in the Rayleigh-Jeans tail imply particle growth toward the millimeter wavelengths of the observations (D'Alessio et al. 2001). Although most observed disks have had more time for grain growth to proceed, Class 0 sources also show evidence of grain growth (D'Alessio et al. 2001). Alternatively, these objects could be optically thick at millimeter wavelengths, which would mimic the effects of grain growth.

Using these opacity laws we see that cooling proceeds with a different functional form in the optically thick and optically thin limits. In the optically thick regime, $f(\tau) \approx \tau = (\kappa_\beta T^\beta \Sigma)/2$, which when combined with equation (4) indicates that to fragment, the disk must have temperature

$$T > \left(f_t f_q^2/2\right)^{1/(2-\beta)} \left(\Omega^3 \kappa_\beta\right)^{1/(2-\beta)} \tag{13}$$

for $\beta \neq 2$. In the special case $\beta = 2$, the fragmentation constraint is temperature-independent, as discussed in §4.2.1.

In the optically thin regime, $f(\tau) = 1/\tau = 2/(\kappa_\beta T^\beta \Sigma)$. The cooling time is independent of Σ, so we can rewrite equation (4) as:

$$T > (2f_t)^{1/(3+\beta)} \left(\frac{\Omega}{\kappa_\beta}\right)^{1/(3+\beta)}, \tag{14}$$

Fig. 1 shows the dependence of the cooling time on disk temperature for each opacity law at two different radii. Since fragmentation can only occur when $\Omega t_{\rm cool} < \zeta$, the minimum temperatures at which fragmentation is allowed are specified by the intersection of the cooling curves with the $\Omega t_{\rm cool}$ boundary.

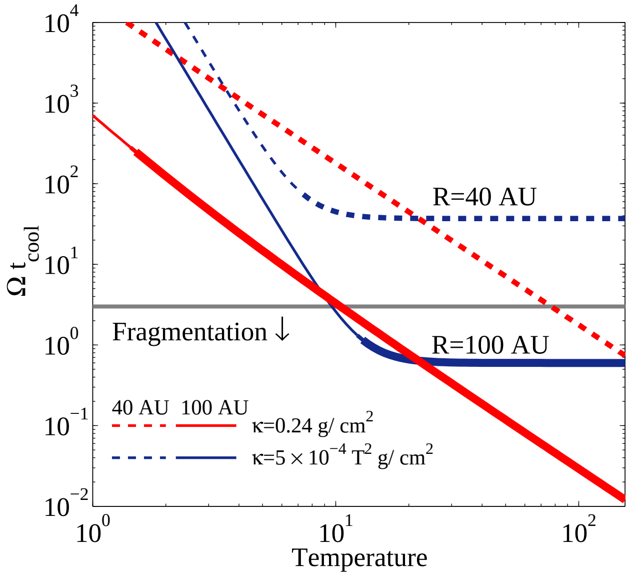

Fig. 1.— The disk cooling time as a function of tempertature for different opacity laws at radii of 40 AU (dashed) and 100 AU (solid). The cooling time is calculated assuming that $Q = 1$. The temperature independent (large grain) opacity law is shown in red, while the ISM opacity law: $\kappa \propto T^2$ is shown in blue. The line thickness indicates the optical depth regime. When lines drop below the critical cooling time (grey), disk fragmentation can occur. The bend in the ISM opacity curve indicates that in the optically thick regime, the cooling time becomes constant as a function of temperature.

4.2.1. *Small grain opacity law*

For an optically thin disk with $\beta = 2$, equation (14) implies that in order to fragment, the disk must have temperatures in excess of:

$$T > T_{\rm thin} = 9\mathrm{K}\left(\frac{\mathrm{r}}{100\ \mathrm{AU}}\right)^{-3/10} \left(\frac{\kappa_2}{5\times10^{-4}\mathrm{cm}^2/\mathrm{g}}\right)^{-1/5} \left(\frac{M_*}{1.5M_\odot}\right)^{1/10} \tag{15}$$

Colder disks, even when optically thin, cannot cool quickly enough to fragment.

For an optically thick disk, $\beta = 2$ turns out to be a special case: the temperature dependence drops out of equation (13), giving instead a critical radius beyond which fragmentation can occur, independent of the disk temperature:

$$r \gtrsim 70\ \mathrm{AU}\left(\frac{M_*}{1.5M_\odot}\right)^{1/3} \left(\frac{\kappa_2}{5\times10^{-4}\mathrm{cm}^2/\mathrm{g}}\right)^{-2/9}. \tag{16}$$

Matzner & Levin (2005) first pointed out the existence of a minimum critical radius for fragmentation. In Fig. 2 we illustrate how the two fragmentation criteria create a radius rather than temperature cutoff. At radii larger than the critical radius defined above, any $\Sigma - T$ combination which gives $Q \leq 1$ will fragment (so long as the opacity law remains valid). At smaller radii, no combination of Σ and T which gives $Q \leq 1$ will fragment because the disk cannot simultaneously satisfy the cooling time criterion.

4.2.2. *Large grain opacity law*

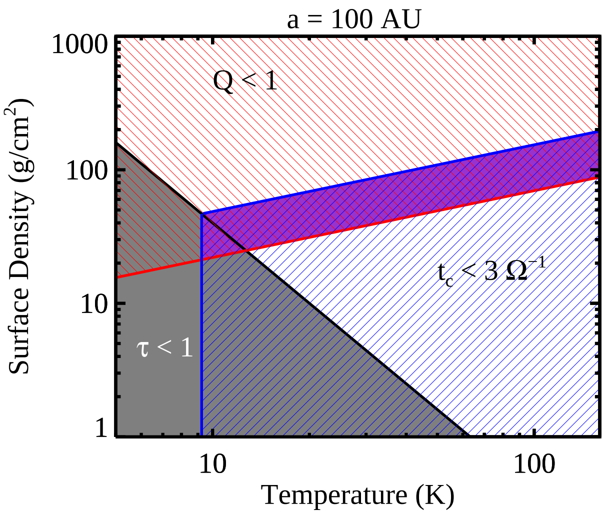

FIG. 2.— Fragmentation can only occur in the region of parameter space indicated by the overlapping hashed regions for ISM opacities at radii of 100 AU. The upper, shaded region (red) shows where Toomre's parameter $Q < 1$. The lower, shaded region (blue) indicates where $t_{\rm cool} \leq 3\Omega^{-1}$. At radii less than 70 AU, fragmentation is prohibited because the two regions no longer overlap. That the boundaries of these regions are parallel lines reflects the $\kappa \propto T^2$ form of the ice-grain-dominated opacity at low temperatures.

As shown in Fig. 1, for the large grain opacity, $\tau > 1$ for all relevant temperatures. In this case equation (13) requires:

$$T > 65K \left(\frac{M_*}{1.5M_\odot}\right)^{3/4} \left(\frac{r}{43\ \mathrm{AU}}\right)^{-9/4} \left(\frac{\kappa_0}{0.24\ \mathrm{cm}^2/\mathrm{g}}\right)^{1/2}. \quad (17)$$

The corresponding minimum mass for these temperatures is:

$$M_{\rm min} = 13 M_{\rm Jup} \left(\frac{M_*}{1.5M_\odot}\right)^{5/8} \left(\frac{r}{43\mathrm{AU}}\right)^{-15/8} \left(\frac{\kappa_0}{0.24\mathrm{cm}^2/\mathrm{g}}\right)^{3/4}. \quad (18)$$

We have scaled equations (17) and (18) to an effective critical radius for this opacity law. Although fragmentation can occur inside 43 AU at sufficiently high temperatures, the fragments exceed $13M_{\rm Jup}$, making it irrelevant for planet formation. Smaller values of κ_0 move this boundary inward, allowing for fragmentation into lower mass objects at smaller radii, although the scaling with opacity is shallow. Grain growth to larger sizes could plausibly reduce κ_0. If, for example, grains dominating the disk opacity have grown up to 1mm without altering the dust-to-gas ratio, equation (12) implies that in the geometric optics limit $\kappa_0 = 0.072\mathrm{cm}^2/\mathrm{g}$. In this case, the minimum radius is pushed inward to 26 AU (see also Nero & Bjorkman 2009).

Thus far we have determined the minimum masses allowed as a function of radius with the temperature as a free parameter. We now calculate actual disk temperatures, which at large radii are typically higher than the minima. In this case we must evaluate fragment masses using equation (6).

4.3. *Initial fragment masses with astrophysical disk temperatures*

To consider the case favorable to GI planet formation, we consider the lowest plausible disk temperatures in order to minimize the fragment masses. We estimate the disk temperature using the passive flared disk models of Chiang & Goldreich (1997). This model likely underestimates the temperatures in actively accreting systems because it ignores significant "backheating" from the infall envelope and surrounding cloud (Chick & Cassen 1997; Matzner & Levin 2005). Although viscous heating will also contribute to the temperature, we ignore its modest contribution to obtain the lowest reasonable temperatures and fragment masses. Disk irradiation dominates over viscous heating in this regime (see Appendix B).

We consider the inner region where the disk is optically thick to blackbody radiation. In this regime, the temperature of a flared disk in radiative and hydrostatic equilibrium is:

$$T_m = \left(\frac{\alpha_F}{4}\right)^{1/4} \left(\frac{R_*}{r}\right)^{1/2} T_* \propto L^{2/7} r^{-3/7} \quad (19)$$

where α_F measures the grazing angle at which starlight hits the disk; α_F is dependent on the degree of disk flaring measured at the height of the photosphere (Chiang & Goldreich 1997). Grain settling may reduce the height of the photosphere (set here to 4 times the scaleheight) and thus α_F. For the standard radiative equilibrium model, the disk flaring scales approximately as $H/r \propto r^{2/7}$. We shall find when we calculate the disk temperature that a gravitationally unstable disk remains optically thick, justifying the use of this formula.

To estimate the the stellar luminosity we use the stellar evolution models of Krumholz & Thompson (2007), which include both nuclear burning and accretion energy. The accretion luminosity depends on both the current accretion rate and the accretion history (in so far as it effects the stellar radius), so we obtain the lowest luminosity estimates by allowing the star to accrete at a constant, low accretion rate. We use the stellar luminosity after accreting to 90% of its current mass (or $1.35M_\odot$ assuming roughly 10% is still in the disk). We choose an accretion rate of $10^{-7}M_\odot$/yr as a lower bound because a star accretingat a lower accretion rate throughout its history has a formation timescale that is too long. Accretion rates an order of magnitude larger give comparable luminosity (when the star has only reached $1.35M_\odot$) to the present day luminosity of $5L_\odot$ (Marois et al. 2008). Lowering the accretion rate below this value does not significantly lower the stellar luminosity because the accretion energy contribution is small.

The luminosity calculated for an accretion rate of $10^{-7}M_\odot$/yr translates to temperatures of:

$$T \approx 40\ \mathrm{K} \left(\frac{r}{70\ \mathrm{AU}}\right)^{-3/7}, \quad (20)$$

which we shall use as our fiducial temperature profile. In the outer regions of the disk where fragmentation is allowed, the disk temperatures are of order $30 - 50$K. These temperatures exceed the minimum threshold for fragmentation, and so the mass of fragments will be set by equation (6).

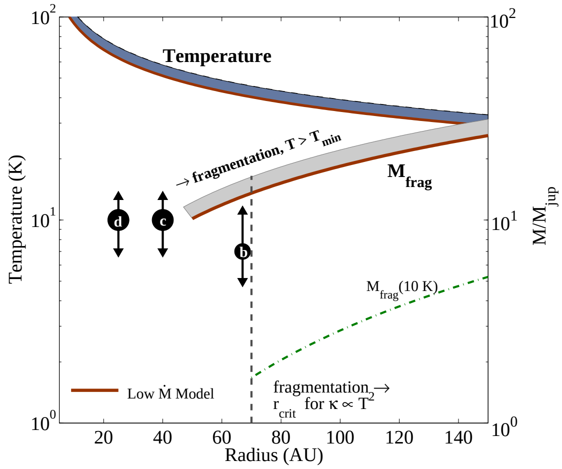

FIG. 3.— Depiction of the current configuration of HR 8799 and formation constraints for realistic disk temperatures. We show the lowest expected irradiated disk temperatures (blue) and corresponding fragment masses (grey), as a function of radius. The lower bound on both regimes (burgundy) is set by the irradiation model described in §4.3, with $\dot{M} = 10^{-7} M_\odot$/yr. The upper boundary is set by the current luminosity of HR 8799, $\sim 5L_\odot$. The green dashed-dotted line shows the mass with disk temperatures of 10 K, a lower limit provided by the cloud temperature. The vertical line shows the critical fragmentation radius for the ISM opacity law; fragmentation at smaller radii requires grain growth. Fragment masses are shown for radii at which the irradiation temperatures are high enough to satisfy the cooling time constraint of equation (17). At smaller radii, fragmentation is possible at higher disk temperatures, but the resulting fragments have correspondingly higher masses, and planet formation is not possible.

Other analytic and numerical models of stellar irradiation predict temperatures in agreement with or higher than our estimate. (Rafikov & De Colle 2006; Offner et al. 2009). Similarly, models of disks in Ophiuchus have similar temperatures for 1 Myr old stars of lower mass (and thus luminosity), implying that our model temperatures are low, though not unrealistic (Andrews et al. 2009).

In Fig. 3 we illustrate the constraints on fragment masses from this irradiation model, calculated using equation (6). We show the fiducial disk temperatures of equation (20), along with temperatures consistent with luminosities up to the present-day luminosity. For our fiducial opacity laws, the expected fragment masses are only marginally consistent with GI planet formation—fragments form near the upper mass limit of $13M_{\rm Jup}$. Lower opacities produced by grain growth might allow fragmentation into smaller objects at closer radii. Whether grain-growth has proceeded to this extent in such young disks is unclear.

5. GROWTH OF FRAGMENTS AFTER FORMATION

For realistic disk temperatures, it is conceivable that fragments may be born at several $M_{\rm Jup}$. We now consider their subsequent growth, which may increase their expected mass by an order of magnitude or more.

The final mass of a planet depends sensitively on numerous disk properties (effective viscosity, column density, scaleheight) and the mass of the embedded object. In order to constrain the mass to which a fragment will grow, we can compare it to two relevant mass scales: the disk isolation mass and the gap opening mass.

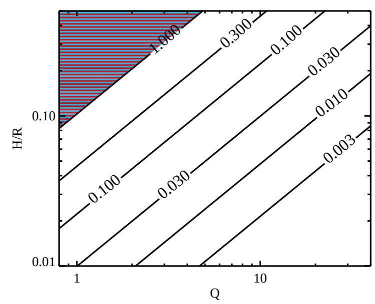

FIG. 4.— Contours of the ratio of planetary isolation mass to stellar mass as a function of Toomre's Q and the disk aspect ratio H/r, illustrating that the isolation mass is always large in unstable disks. For disks with higher Q's consistent with core accretion models, the isolation mass remains small. The shaded region indicates where the isolation mass exceeds the stellar mass.

Halting the growth of planetary mass objects is a relevant problem independent of the formation mechanism. However the GI hypothesis requires that the disk is (or was recently) sufficiently massive to have $Q \sim 1$, implying that the disk is actively accreting. The core accretion scenario does not face this restriction.

5.1. *Isolation Mass*

We estimate an upper mass limit for fragments by assuming that they accrete all of the matter within several Hill radii:

$$M_{\rm iso} \approx 4\pi f_H \Sigma R_H r. \qquad (21)$$

Here $f_H \sim 3.5$ is a numerical constant representing from how many Hill radii, $R_H = r(M_{\rm iso}/3M_*)^{1/3}$, the planet can accrete (Lissauer 1987).

It is instructive to compare the ratio of the isolation mass to the stellar mass:

$$\frac{M_{\rm iso}}{M_*} = 4.6 f_H^{3/2} Q^{-3/2} \left(\frac{H}{r}\right)^{3/2}. \qquad (22)$$

We find that large isolation masses are always expected in gravitationally unstable disks. Fig. 4 illustrates the scaling of equation (22) with Q and the disk aspect ratio, H/r. For our fiducial temperature profile, $H/r \approx 0.09$ at 70 AU. For low values of Q and comparable H/r, the isolation mass exceeds 10% of the stellar mass. For the ideal disk values cited above (equation 16), the isolation mass is:

$$M_{\rm iso} \approx 400 M_{\rm Jup} \left(\frac{r}{70\ \rm AU}\right)^{6/5} \left(\frac{M_*}{1.5 M_\odot}\right)^{1/10}. \qquad (23)$$

This mass is nearly two orders of magnitude larger than the minimum mass. Growth beyond the isolation mass is possible either through mergers or introduction of fresh material to accrete by planet migration or disk spreading.

Objects which grow to isolation mass cannot be planets, and so we turn to mechanisms that truncate fragment growth below the isolation mass.

5.2. *Gap opening mass*

Massive objects open gaps in their disks when gravitational torques are sufficiently strong to clear out nearby gas before viscous torques can replenish the region with new material. (Lin & Papaloizou 1986; Bryden et al.

1999). The gap width is set by the balance between the two torques:

$$\frac{\Delta}{r} = \left(\frac{f_g q^2}{3\pi\alpha}\frac{r^2}{H^2}\right)^{1/3}, \tag{24}$$

where Δ is the gap width, $f_g \approx 0.23$ is a geometric factor derived in Lin & Papaloizou (1993), q is the planet to star mass ratio, and α is the Shakura & Sunyaev (1973) effective viscosity. This can be used to derive the standard minimum gap opening mass by requiring that $\Delta > H$:

$$\begin{aligned} q > & \left(\frac{H}{r}\right)^{5/2}\sqrt{\frac{3\pi\alpha}{f_g}} \qquad (25)\\ \approx & \, 4\times10^{-3}\left(\frac{\alpha}{0.1}\right)^{1/2}\left(\frac{T}{40\text{ K}}\right)^{5/4}\left(\frac{r}{70\text{ AU}}\right)^{5/4} \\ & \left(\frac{M_*}{1.5\ M_\odot}\right)^{-5/4}. \end{aligned}$$

Gap opening requires $\Delta > R_\mathrm{H}$ and $R_\mathrm{H} > H$. The latter requirement is automatically satisfied for fragments formed by GI.

While the effects of GI are often parameterized through an α viscosity, Balbus & Papaloizou (1999) have pointed out that α, a purely local quantity, may not adequately describe GI driven transport, which is inherently non-local. Lodato & Rice (2005) have shown that for sufficiently thin disks, the approximation is reasonable: in order to form objects of planetary mass, the disk must be relatively thin and at least marginally within this limit. However, even in this thin-disk limit, it is not clear that GI driven torques will exactly mimic viscous ones at gap-opening scales.

Equation (25) implies that the gap opening mass is less than or equal to the fragment mass for effective viscosities consistent with GI. We use $\alpha \approx 0.1$, as this is consistent with active GI (Gammie 2001; Lodato & Rice 2005; Krumholz et al. 2007). If the local disk viscosity is lower, fragments will always form above the gap-opening mass.

5.2.1. *Gap-opening starvation mass*

Gap-opening slows accretion onto the planet, but does not starve it of material completely. Accretion rates through gaps remain uncertain for standard core accretion models, and numerical models are not available for accretion onto the distended objects formed through GI fragmentation. Nevertheless simulations of accretion through gaps in low viscosity disks (Lubow et al. 1999) demonstrate that accretion is slower through larger gaps, and this qualitative conclusion likely remains valid as long as gaps form.

Analogous to the isolation mass, we consider a “gap starvation mass” that is related to the ratio of the gap width to planet Hill radius. Rewriting equation (24) we find the ratio of gap width to Hill radius is:

$$\frac{\Delta}{R_H} = \left(\frac{f_g q}{\pi\alpha}\frac{r^2}{H^2}\right)^{1/3} \tag{26}$$

Note that $\Delta > R_H$ recovers the canonical gap opening estimate appropriate for Jupiter: $q > 40\nu/(r^2\Omega)$, modulo order unity coefficients (cf. Crida et al. 2006).

If we make the simplifying assumption that gap accretion terminates when the gap width reaches a fixed number of Hill radii, f_S, we can calculate a gap starvation mass. We expect that for a gap to truncate accretion, $f_S \gtrsim f_H = 3.5$, the width in Hill radii used to calculate the isolation mass (§5.1). Terminating accretion is an unsolved problem for Jupiter in our own solar system, and so to provide a further constraint on f_S, we refer to the numerical simulations of Lissauer et al. (2009) (See also D'Angelo et al. (2003) for a detailed explanation of the numerical work). Their runs 2l and 2lJ exhibit asymptotic mass growth after $\sim$2.5 Myr for a constant-mass, low viscosity ($\alpha = 4\times10^{-4}$) disk under conditions appropriate to the formation of Jupiter. Using equation (26), we solve for the width of the gap generating this fall-off in accretion rate and find $f_S = \Delta/R_H \sim 5$. The need for an extremely large and well-cleared gap reflects the integrated effects of low-level accretion through the gap and onto the planet over the disk lifetime of a few Myr. Even a slow trickle of material onto the planet can contribute to significant growth.

Using $f_S = 5$, the gap-opening starvation mass for HR 8799 scaled to both the simulated solar-system viscosity and to the expected GI viscosity is:

$$\begin{aligned} M_\mathrm{starve} \approx & \, 8M_\mathrm{Jup}\left(\frac{\alpha}{4\times10^{-4}}\right)\left(\frac{\Delta}{5R_H}\right)^3 \qquad (27)\\ & \left(\frac{T}{40\text{ K}}\right)\left(\frac{r}{70\text{ AU}}\right) \\ \approx & \, 2000M_\mathrm{Jup}\left(\frac{\alpha}{0.1}\right)\left(\frac{\Delta}{5R_H}\right)^3 \\ & \left(\frac{T}{40\text{ K}}\right)\left(\frac{r}{70\text{ AU}}\right). \end{aligned}$$

In order to limit growth to planetary masses, the effective viscosity must be two orders of magnitude below that expected in GI unstable disks, roughly $\alpha \sim 10^{-3}$. More restrictively this requires that other local transport mechanisms such as the MRI be weaker than currently predicted by simulations—they produce $\alpha \sim 10^{-2}$ at least in disks with a net magnetic flux (Fleming et al. 2000; Fromang et al. 2007; Johansen et al. 2009). Fig. 5 illustrates the scaling of gap starvation mass with radius for several values of α. It appears that active disks face severe obstacles in producing planetary mass objects unless the disk disappears promptly after their formation. Although we expect fragmentation to make the disk more stable by lowering the local column density, there is little reason to expect a recently massive disk to be so quiescent.

5.2.2. *Planet starvation through gap overlap*

Although it is unlikely that the disk will fragment into a sufficiently large number of planetary mass objects to completely deplete the disk of mass (Stamatellos & Whitworth 2009), the formation of multiple fragments simultaneously may limit accretion through competition for disk material by opening overlapping gaps. The current separation between the planets is such that gaps larger than roughly three Hill radii overlap, so depending on their migration history, this could limit growth

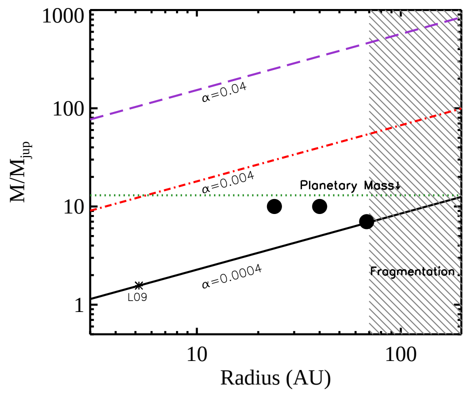

FIG. 5.— The gap starvation mass as a function of disk radius. We show curves for several values for α, and indicate the planetary mass regime, and the region in which disk fragmentation is likely. We use $f_S = 5$, scaled to simulation 2lJ of Jupiter formation in Lissauer et al. (2009) (labeled L09 in the figure). The radial scaling is derived assuming $H/r \propto r^{2/7}$. For the low viscosity case, we normalize the scale height to Jupiter at 5.2 AU in a 115 K disk for comparison with L09. For the higher viscosities, we normalize the disk scale height to the lowest expected temperatures (equation 20). For comparison we show the HR 8799 planets as black circles.

(see also §7). From equation (27) we see that if gaps are forced to be smaller by a factor of two due to competition with another planet, the expected masses are decreased by a factor of 8. This effect would imply that fragments in multiple systems should be lower in mass. Note that when simulated disks fragment into multiple objects simultaneously, they generally have orbital configurations like hierarchical multiples rather than planetary systems (Stamatellos & Whitworth 2009; Kratter et al. 2010). Whether the same conditions required to limit fragment growth—reduced disk mass and/or viscosity after fragmentation—can allow the retention of a planetary system of fragments remains to be simulated.

5.3. *Disk dispersal as a means to limit fragment growth*

Mechanisms for gas dispersal such as photoevaporation may be necessary to stunt planetary growth, even for models of Jupiter in our own solar system (Lissauer et al. 2009). Dissipation timescales for A-star gas disks are thought to be short, less than 2-3 Myr (Carpenter et al. 2006), which could halt growth before the gap-opening starvation mass is achieved. Radiative transfer models such as Gorti & Hollenbach (2009) have calculated that photoevaporation by the central star will become important at radii of ~ 100 AU around one Myr for an A star (see also Ercolano et al. 2009). This timescale coincides with the expected fragmentation epoch, and may aid in shutting off accretion both onto the disk, and onto the planets.

6. GI PLANET FORMATION IN THE CONTEXT OF STAR FORMATION

We now consider how the disk can reach the fragmentation conditions described in §3 in the context of a model for star formation. Due to the effects of infall onto the disk, we find that planet formation via GI can only occur when the fragmentation epoch is concurrent with the end of the main accretion phase, as the protostar transitions from a Class I to a Class II object (Andre & Montmerle 1994). Fragmentation at earlier times leads to the formation of more massive companions, while fragmentation at later times is unlikely because disks are too low in mass (Andrews et al. 2009).

6.1. *Ongoing accretion and the formation of binaries and multiples*

Because the cooling time constraint is easily satisfied for expected disk temperatures, disks are likely driven to fragmentation by lowering Q. Even when Q is above the threshold for fragmentation, torques generated by self-gravity (e.g. spiral arms) can drive accretion. When the infall rate onto the disk is low, a self-gravitating disk regulates its surface density and hence Q so that the torques are just large enough to transport the supplied mass down to the star, thereby avoiding fragmentation. However, GI cannot process matter arbitrarily quickly because the torques saturate. Thus the disk will be driven toward fragmentation if the infall rate becomes too high. This critical accretion rate is a function of disk temperature:

$$\dot{M}_{\rm crit} \approx \frac{3c_s^3\alpha_{\rm sat}}{GQ} \tag{28}$$

(Gammie 2001; Matzner & Levin 2005). Numerical simulations show that GI saturates at $\alpha_{\rm sat} \sim 0.3-1$ (Gammie 2001; Krumholz et al. 2007; Lodato & Rice 2005; Kratter et al. 2010). If the infall rate onto the disk exceeds $\dot{M}_{\rm crit}$ the disk can no longer regulate the surface density to keep Q just above unity, and fragmentation will occur. Because the conversion of accretion energy to thermal energy at large radii is inefficient, disks cannot restabilize through heating to arbitrarily high accretion rates (Kratter & Matzner 2006).

If fragmentation occurs due to rapid accretion, as described above, it is difficult to limit subsequent fragment growth. As demonstrated in §5.2.1, gap opening does not limit accretion efficiently when the effective viscosity, in this case, $\alpha_{\rm sat}$, is high.

A more general barrier to making small fragments during rapid infall is the large reservoir of material passing by the fragment as star formation proceeds (Bonnell & Bate 1994). The specific angular momentum, j, of accreting disk material typically increases with time (modulo small random fluctuations in a turbulent core), landing at a circularization radius, $r_{\rm circ} = j^2/GM_*$, which is larger than the fragment's orbit. Since the fragment's Hill radius is roughly 10% of the disk radius, newly accreted material undergoes many orbits in the fragment's sphere of influence as it tries to accrete onto the central star, and some fraction will accrete onto the fragment itself. This process is less efficient if global GI modes drive fragments to smaller radii, because their Hill radii shrink. However, growth to stellar (or sub-stellar) masses may still occur as long as migration timescales are not faster than accretion timescales. The latter scenario implies that the disk cannot fragment at early times and still reproduce a single-star system like HR 8799.

The trend toward continued growth and even mass equalization following disk fragmentation is observed in numerous simulations with ongoing accretion (Bonnell & Bate 1994; Bate 2000; Matsumoto & Hanawa 2003; Walch et al. 2009; Kratter et al. 2010; Krumholz et al.

2009). Simulations of planet formation by GI with ongoing accretion also illustrate this behavior (Boley 2009).

Consequently, if the HR 8799 planets formed by GI, they must have fragmented at the tail end of accretion from the protostellar core onto the disk. Most likely, this requires that the protostellar core have properties such that its infall rate reaches the critical value in equation (28) just as the core is drained of material. If this coincidence in timing does not occur, fragmentation produces a substellar, rather than a planetary, companion. Whether fragmentation at this epoch can produce non-heirarchical orbits like HR 8799 remains to be explored.

6.2. *Reaching Instability in the absence of infall: FU Orionis outbursts*

Driving the disk unstable with external accretion corresponds to excessive growth of fragments. It is therefore tempting to consider mechanisms to lower Q through disk cooling, while holding the column density fixed. Because the disks are dominated by irradiation, changes in the viscous dissipation due to accretion are unlikely to affect a significant temperature change, and so lowering Q requires order of magnitude changes in the stellar luminosity due to the weak scaling of $T \propto L^{2/7}$. FU Orionis type outbursts (Hartmann & Kenyon 1996) can cause rapid changes in luminosity. To result in planetary mass fragments, the luminosity drop following an FU Ori outburst would need to reach at least the minimum luminosity used in equation (20) on timescales shorter than an outer disk dynamical time.

The accretion of GI formed embryos onto the protostar is a proposed source of the outbursts (Vorobyov & Basu 2006). If this occurs, perhaps a final generation of gravitational fragments, the so-called "last of the Mohicans" (Gonzalez 1997), would remain as detectable companions. Although the lack of infall in this scenario might ease the gap opening constraints, fragments face all of the other difficulties discussed above in remaining low in mass.

7. MIGRATION IN A MULTI-PLANET SYSTEM

A final consideration for making wide orbit planets through GI is their subsequent migration history. If formed via GI, the HR 8799 planets had to migrate inward to their current locations. There is an independent reason to believe that migration did in fact take place in this system. As discussed by Fabrycky & Murray-Clay (2008), the long term stability of HR 8799 requires a resonant orbital configuration, most plausibly a 4:2:1 mean motion resonance, which likely resulted from convergent migration in the protoplanetary gas disk. We demonstrate below that while this history is plausible, it requires special disk conditions.

Inward Type II migration (appropriate for gap opening planets) is expected at formation, because the fragmenting region is within the part of the disk accreting onto the star. Type II migration of a single planet occurs on the disk viscous timescale:

$$\tau_\nu \approx \frac{r^2}{\nu} = \frac{r^2\Omega}{\alpha c_s^2} \tag{29}$$

$$\approx 0.4\,\mathrm{Myr}\left(\frac{\mathrm{r}}{70\ \mathrm{AU}}\right)^{13/14}\left(\frac{\mathrm{M}_*}{1.5\mathrm{M}_\odot}\right)^{1/2}\left(\frac{\alpha}{0.1}\right)^{-1} \tag{30}$$

where we have used $\nu = \alpha c_s^2/\Omega$. This timescale is short enough to allow substantial migration during the lifetime of the gas disk.

If fragments migrated inward independently, they could never become captured in resonance, as the innermost planet would migrate farther and farther from its neighbor. However, as shown in §5, planetary gaps may overlap if they are within several Hill radii of each other, comparable to the current separations between the HR 8799 planets. This overlap alters the torques felt by, and thus the migration of, the planets. As shown by Kley (2000), if multiple planetary gaps interact, convergent migration is possible. Gap interaction allows an outer planet to shield an inner planet from the material, and thus torques, of the outer disk, slowing or halting its inward migration and allowing the outer planet to catch up. This mechanism is invoked by Lee & Peale (2002) to generate convergent migration and resonance capture in the planets orbiting GJ 876.

Under the assumption that gap overlap allows convergent migration and resonance capture, we now ask: What is the overall direction of the subsequent migration? We note that if the planets migrated a substantial distance after resonance capture, eccentricity damping by the gas disk was likely necessary (c.f., Lee & Peale 2002). We do not consider eccentricity damping further here. Once two planets are caught in mean-motion resonance, the torque on an individual planet from the gas disk can cause both planets to migrate, with angular momentum transfer mediated by the resonance. Masset & Snellgrove (2001) (see also Crida et al. 2009) have argued that the torque imbalance on a pair of gap-opening resonant planets can even reverse the direction of migration, although this relies on a significant difference between planet masses.

Nevertheless, understanding the planets' overall migration requires understanding how overlapping gaps alter the torque balance on the group of planets. Guided by our interest in clean gaps that limit the growth of planets (§5–6), we consider the following simplified problem.

We imagine that the three planets have cleared, and are embedded in, a single large gap which is sufficiently clean that any disk gas passing through is dynamically unimportant. Because the system is locked in a double mean-motion resonance, we assume that an imbalance in the torques acting on the two edges of the gap can cause all three planets to migrate. We can then ask: is there a sufficient flux of angular momentum from the outer disk to cause the planets to migrate inward with the viscous accretion of the disk?

When in the 4:2:1 resonance, the total angular momentum of the planets is roughly $2.4M_p\Omega_p r_p^2$, where we have assumed that the planets are roughly equal in mass, M_p, while r_p and Ω_p are the separation and Keplerian angular velocity of the outermost planet. The angular momentum flux from the outer disk is large enough to move the planets on a viscous time r_p^2/ν when:

$$\dot{M}\Omega_p r_p^2 \gtrsim 2.4M_p\Omega_p r_p^2(\nu/r_p^2)\ , \tag{31}$$

where $\dot{M} = 3\pi\Sigma\nu$ is the mass flux through radius r_p.

Using the above inequality, we can calculate a critical disk surface density at the current location of the outer

planet such that the disk can push the planets inward:

$$\Sigma \gtrsim \frac{M_p}{4r_p^2}. \tag{32}$$

For $M_p = 10M_{\rm Jup}$ and our fiducial disk temperatures (equation 20), this constraint is always satisfied when $Q = 1$. The disk is unable to cause inward migration when $\Sigma \lesssim 4{\rm g/cm^2}$ at 70 AU, which is equivalent to $Q \sim 20$.

If the planets do share a clean common gap, a large fraction of the disk would be effectively cleared of gas while a massive outer disk is still present. A similar mechanism has been invoked to explain transitional disks, which contain holes at radii of a few tens of AU and smaller (Calvet et al. 2002). Transport of disk gas through a less well-cleared gap could substantially alter this picture.

In summary, it is possible to envision a scenario in which the HR 8799 planets migrate inward to their current locations in such a way that their orbits converge, allowing resonance capture. This scenario is consistent with other constraints on GI planet formation: shortly after formation, the disk must have low accretion rates and decline in mass in order to (a) limit the growth of fragments, (b) allow for large, overlapping gaps.

More stringent constraints will require future work on migration in gravitationally unstable disks, particularly in the presence of multiple planets massive enough to clear large, overlapping gaps.

8. CURRENT OBSERVATIONAL CONSTRAINTS

While there is a regime of parameter space in which planet formation is possible, typical conditions produce more massive ($> 13M_{\rm Jup}$) companions. If GI fragmentation ever forms planets, then fragmentation should typically form more massive objects. Consequently, these planets would constitute the low mass tail of a distribution of disk-born companions. If the mass distribution is continuous there should be more sub-stellar companions than planets at comparable distances of 50-150 AU. Observing this population is a strong constraint on the formation mechanism, but current data are insufficient to draw conclusions.

Zuckerman & Song (2009) have compiled the known sub-stellar companions in this range of radii to date. This range of separations falls beyond the well-established inner brown dwarf "desert" (McCarthy & Zuckerman 2004), and has not been well probed due to observational difficulties at these low mass ratios. Note that the overall dearth of brown dwarf companions to solar mass stars is not a selection effect (Metchev & Hillenbrand 2009; Zuckerman & Song 2009).

We illustrate the observational constraints by plotting the companions from Zuckerman & Song (2009) along with the known exoplanets compiled by the Exoplanet Encyclopedia[2] as a function of mass ratio and projected separation in Fig. 6a, and as a function of minimum fragment masses and fragmentation radii in Fig. 6b. We compare these with the HR 8799 planets, Fomalhaut b, and the solar system giants. We distinguish between stars of different masses because disk fragmentation becomes more likely for higher mass stars (Kratter et al. 2008).

At present, neither the population of substellar companions nor the population of exoplanets is continuous out to HR 8799. Many selection biases are reflected in Fig. 6, and these gaps in particular may be due to selection effects; resolution and sensitivity make it difficult to detect both wide orbit planets, and close-in low mass brown dwarfs. We note that while there are actually fewer planets at distances less than 1 AU, the cutoff above 5 AU is unphysical. There is not yet any indication of an outer cut off radius in the exoplanets: if they continued out to larger separation, the distribution would easily encompass the HR 8799 and Fomalhaut systems.

Data from ongoing surveys like that which found HR 8799 are necessary to verify the true companion distribution as a function of mass and radius. If these planets are formed through GI, we would expect observations to fill in the gap between HR 8799 and higher mass ratio objects to show a continuous distribution. If these planets are formed via core accretion, than observations may fill in the plot on the opposite side of HR 8799, occupying a region of parameter space for which neither core accretion nor GI is currently a successful formation mechanism.

9. SUMMARY

We have demonstrated that while GI-driven fragmentation is possible at wide distances from A stars, fragment masses typically exceed the deuterium burning "planet" limit. In contrast, the formation of sub-stellar and stellar companions is more likely because moderate disk temperatures and active accretion onto and through the disk drive disk-born objects to higher masses.

If the HR 8799 planets did form by GI, the following criteria had to be met:

1. Fragments should form beyond $40 - 70$ AU: inside of this location the disk will not fragment into planetary mass objects even if $Q \lesssim 1$. Grain growth is required for fragmentation at the lower end of this range.

2. Temperatures must be colder than those of typical disks to limit the initial fragment masses.

3. The disk must be driven unstable at a special time: infall onto the disk must be low, but the disk must remain massive (e.g. the end of the Class I phase). The disk must only become unstable to fragmentation at this point because earlier episodes of instability should lead to sub-stellar or stellar companion formation.

4. The subsequent growth of fragments must be limited through efficient gap clearing necessitating low disk viscosity or early gap overlap. Disk dispersal via photoevaporation may also be necessary.

5. The three fragments must form at the same epoch separated by several Hill radii, implying that the entire outer disk becomes unstable simultaneously.

6. Migration must be convergent. This likely requires the gaps of the planets to overlap so as to starve

[2] October 2009, http://exoplanet.eu, compiled by Jean Schneider

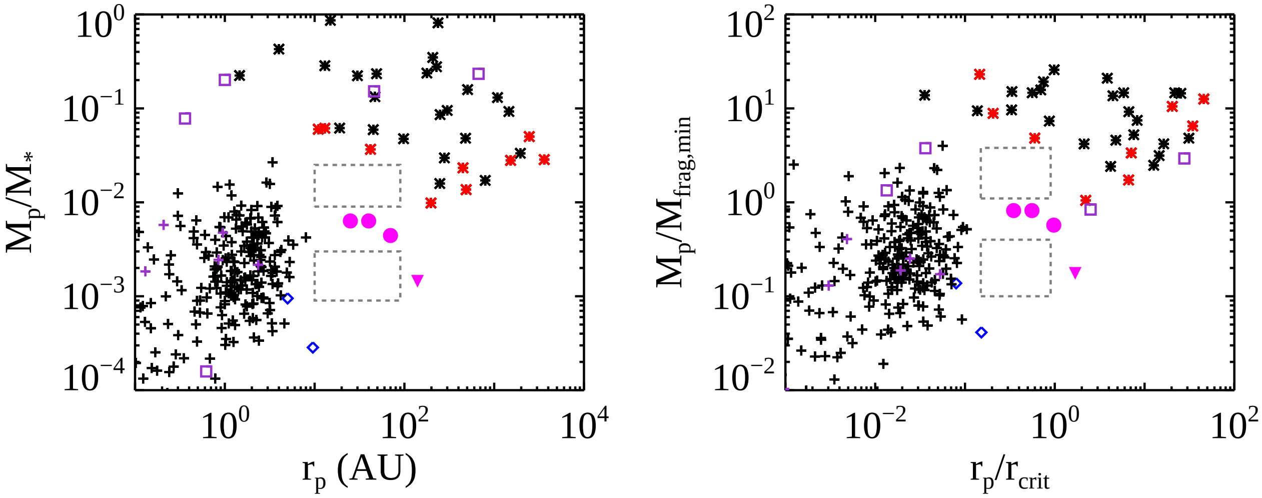

FIG. 6.— (Left) Known substellar companions (stars) and planets (plusses) as a function of mass ratio and projected separation. The three objects in the HR 8799 system are shown by pink circles, and a pink triangle denotes the upper-limit mass ratio for Fomalhaut b based on the dynamical mass estimate of Chiang et al. (2009). Grey squares indicate the gap regions. Ongoing surveys are necessary to determine whether there is a continuous distribution between Jupiter/Saturn (blue diamonds) and HR 8799, or HR 8799 and brown dwarf companions. Planets around very low mass primaries with $M_* = 0.02$–$0.1 M_\odot$ are marked by purple squares. These systems have mass ratios more akin to the substellar companions than to the remainder of the population of planets. Primary masses range from $M_* = 0.02 M_\odot$–$1 M_\odot$ (black) and $M_* = 1 M_\odot$–$2.9 M_\odot$ (red) for substellar companions and from $M_* = 0.1$–$0.4 M_\odot$ (purple) and $M_* = 0.4$–$4.5 M_\odot$ (black) for planets. (Right) The same objects plotted as function of the minimum fragment mass, $M_{\rm frag,min}$ and critical radius, $r_{\rm crit}$. We use equation (16) to calculate $r_{\rm crit}$. For $M_{\rm frag,min}$, we apply equation (6) at radius $r_{\rm crit}$ under the simplified assumption that the disk temperature is set by the stellar luminosity: $L/L_\odot = (M_*/M_\odot)^{3.5}$ for $M_* > 0.43 M_\odot$ and $L \propto M_*^{2.3}$ for lower-mass stars. The temperatures used to calculate fragment masses are not allowed to dip below 20K. Masses below $M_{\rm frag,min}$ are unlikely to result from GI.

the inner most planet of disk material, thereby preventing runaway inward migration.

If these conditions are met, then the planets in HR 8799 could comprise the low-mass tail of the disk-born binary distribution, the runts of the litter. In this case one would expect to find a larger number of brown dwarfs or even M stars in the same regime of parameter space – surrounding A-stars at distances of $50 - 150$ AU.

Ongoing direct imaging surveys of A and F stars will provide a strong constraint on the formation mechanism for this system: if HR 8799 is the most massive of a new distribution of widely separated planets, our analysis suggests that formation by GI is unlikely. On the contrary, the discovery of a population of brown dwarf and M-star companions to A-stars would corroborate formation via disk fragmentation.

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APPENDIX

A. COOLING AND FRAGMENTATION IN IRRADIATED DISKS

Rafikov (2009) has suggested that the cooling time might be altered in an irradiated disk. Here we consider the cooling time for thermal perturbations to a disk, and show that a simple formula [equation (A7)] gives the cooling time for arbitrary levels of irradiation, at least in radiative optically thick disks.

Consider ambient radiation striking an optically thick disk with a normal flux $F_o = \sigma T_o^4 \equiv (3/8)F_{\rm irr}$ (where the numerical factor in the last definition is purely for later convenience). Note that T_o depends both on the irradiation field (from the host star and/or light emitted and reflected from a surrounding envelope) and also on the disk's surface geometry, e.g. flaring angle. By incorporating these variables into T_o we attempt a general calculation. At the photosphere, where the optical depth to the disk's self-emission $\tau = \tau_{\rm phot} \approx 1$, energy balance gives:

$$\sigma T_{\rm eff}^4 \simeq \sigma T_o^4 + F/2\,. \tag{A1}$$

Here, F is the luminous flux from any internal sources of energy dissipation, e.g. viscous accretion, shocks, or the gravitational binding energy released by a collapsing fragment. The factor of two reflects that half of the radiation is emitted from each surface of the disk. Since optical light is absorbed above the IR photosphere, about half the irradiation free streams out before it heats the disk (Chiang & Goldreich 1997). We can absorb this reduction into the definition of T_o.

We assume heat is transferred by radiative diffusion as:

$$\frac{4}{3}\sigma\frac{dT^4}{d\tau} = \frac{F}{2}. \tag{A2}$$

since convection is suppressed by irradiation and may be a negligible correction in any event (Rafikov 2007). Integration from the midplane at $\tau = \tau_{\rm tot}$ and $T = T_{\rm m}$ to the photosphere gives:

$$(4/3)\sigma(T_{\rm m}^4 - T_{\rm eff}^4) = (F/2)(\tau_{\rm tot} - \tau_{\rm phot})\,. \tag{A3}$$

We now drop the subscripts from $\tau_{\rm tot}$ and the midplane $T_{\rm m}$, which we will soon take as the characteristic temperature (ignoring order unity corrections from height averaging). Furthermore we apply equation (A1) and take the $\tau_{\rm phot} \ll \tau_{\rm tot} \rightarrow \tau$ limit (meaning that we don't need to know the precise location of the photosphere) to express

$$F \simeq \frac{8\sigma}{3\tau}(T^4 - T_o^4) = \frac{1}{\tau}\left(\frac{8\sigma T^4}{3} - F_{\rm irr}\right), \tag{A4}$$

which shows that the midplane temperature is controlled by the larger of $F\tau$ and $F_{\rm irr}$.

The cooling timescale to radiate away thermal fluctuations (generated e.g. by GI) is

$$t_{\rm cool} = \frac{\Sigma \delta U}{\delta F} \tag{A5}$$

where a temperature perturbation δT has an excess heat $\delta U \approx c_P \delta T$, and $c_P = (k/\mu)\gamma/(\gamma-1)$ is the specific heat for a mean molecular weight μ and adiabatic index γ. Strongly compressive motions, which are not at constant pressure, will introduce order unity corrections that we ignore.

The excess luminous flux, using equation (A4), is

$$\begin{aligned}\delta F &= \frac{8\sigma}{3\tau}\frac{\delta T}{T}\left[(4-\beta)T^4 + \beta T_o^4\right] \\ &= \frac{32\sigma T^3 \delta T}{3\tau} \times \begin{cases} (1-\beta/4) & \text{if } T \gg T_o, \beta \neq 4 \\ 1 & \text{if } T \simeq T_o \end{cases},\end{aligned} \tag{A6}$$

where $\tau = \kappa\Sigma/2 \propto T^\beta$. The point is that the escaping flux varies by only an order unity factor between the strongly ($T \simeq T_o$) and weakly ($T \gg T_o$) irradiated regimes. Typical grain opacities, $0 < \beta < 2$, ensure the correction is order unity (and also ensure that we can ignore the catastrophic heating that would occur if $\beta > 4$).

Combining equations (A5) and (A6) with the definition of heat capacity we find that the cooling time is simply

$$t_{\rm cool} \approx \frac{3\gamma\Sigma c_s^2 \tau}{32(\gamma-1)\sigma T^4} \times \begin{cases} (1-\beta/4)^{-1} & \text{if } T \gg T_o, \beta \neq 4 \\ 1 & \text{if } T \simeq T_o \end{cases}, \tag{A7}$$

where the isothermal sound speed $c_s = \sqrt{kT/\mu}$.

From this derivation, we see that the cooling time obeys the simple form of equation (A7) for all levels of irradiation — which only introduces an order unity β correction. The cooling time depends on β for weakly irradiated disks because changes to the opacity alter the amount of flux that escapes from the midplane. In highly irradiated disks, opacity changes have little effect because the small difference between the midplane and surface temperatures drives a weak flux.

While increasing the irradiative flux incident on a disk decreases the cooling time by raising T, it will not trigger fragmentation in a $Q \sim 1$ disk, since it increases Q. We will not explore optically thin or convective disks at this time.

We assume that $\Omega t_{\rm cool} < \zeta \sim 3$ is the fragmentation criterion independent of irradiation. When the cooling time is longer, the disk is presumed to enter a state of gravito-turbulence (Gammie 2001). We take this term to mean a quasi-steady state of gravitationally driven turbulence, on scales $\lesssim H$ wherein viscous dissipation of GI turbulence regulates $Q \sim 1$. Thus the cooling time can be translated to a the critical value of α at which gravito-turbulent accretion disks will fragment. While Rice et al. (2005) find that an α-threshold is more robust than one for $t_{\rm cool}$ when the adiabatic index varies, they did not include irradiation, which we contend would reveal that cooling is ultimately the more physical criterion, but the issue is best settled by simulation. The emitted flux with an α-viscosity and $Q \sim 1$ gives

$$F \approx (9/4)\nu\Sigma\Omega^2 \approx 9/(4\pi)\alpha c_s^3 \Omega^2/G \tag{A8}$$

which combined with equation (A4) and equation (A7) without the β correction gives

$$\Omega t_{\rm cool} \approx \left(\frac{\gamma}{\gamma-1}\right)\frac{\Omega\Sigma c_s^2}{4\left(F + F_{\rm irr}/\tau\right)} \approx \left(\frac{\gamma}{\gamma-1}\right)\frac{1}{9\alpha}\left(1 + \frac{F_{\rm irr}}{F\tau}\right)^{-1} \tag{A9}$$

When irradiation is weak enough that $F_{\rm irr} \lesssim F\tau$, we recover the standard $\alpha \gtrsim 1$ criterion for fragmentation (ignoring the accumulated order unity coefficients). However for stronger irradiation with $F_{\rm irr} \gtrsim F\tau$, fragmentation occurs for $\alpha \gtrsim F\tau/F_{\rm irr}$, a lower threshold.

Gravito-turbulent models require modification when $F_{\rm irr} \gtrsim F$, i.e. an even lower level of irradiation than needed to affect the α fragmentation threshold. In this case the disk shows some similarities to isothermal disks, and should have lower amplitude density perturbations, because there is insufficient viscous dissipation to support order unity thermal perturbations (see the related discussion in Rafikov 2009). We note that simulations of isothermal disks do develop GI, exhibit GI-driven transport and fragment (Krumholz et al. 2007; Kratter et al. 2010), but they do not appear particularly turbulent.

B. TEMPERATURE DUE TO VISCOUS HEATING

We now show that viscous heating is relatively unimportant in the outer reaches of irradiated A-star disks (see also section 3 of Rafikov 2009). For an optically thick disk with an ISM opacity law, balancing viscous heating and emitted radiation gives

$$\frac{8}{3\tau}\sigma T^4 \approx \frac{3}{8\pi}\dot{M}\Omega^2 \tag{B1}$$

The solution for the midplane temperature is

$$T \approx \left(\frac{9}{128\pi^2}\frac{\dot{M}\kappa_o\sqrt{k/\mu}}{\sigma G Q_o}\right)^{2/3}\Omega^2 \approx 9\ \mathrm{K}\left(\frac{\dot{M}}{10^{-6}M_\odot/\mathrm{yr}}\right)^{2/3} Q_o^{-2/3}\left(\frac{r}{70\ \mathrm{AU}}\right)^{-3}, \tag{B2}$$

lower than the irradiation temperatures shown in Fig. 3 by more than a factor of four. Note that the surface density falloff

$$\Sigma = \frac{c_s\Omega}{\pi G Q_o} \approx 35\ \mathrm{g/cm^2}\left(\frac{r}{70\ \mathrm{AU}}\right)^{-3}\left(\frac{\dot{M}}{10^{-6}M_\odot/\mathrm{yr}}\right)^{1/3} \tag{B3}$$

is also quite steep for a constant $Q_0 = 1$, viscous disk with the ISM opacity law.

If the disk is optically thin, then the balance between heating and cooling gives:

$$4\tau\sigma T^4 \approx \frac{3}{8\pi}\dot{M}\Omega^2 \tag{B4}$$

$$T \approx \left(\frac{3G\dot{M}Q_o\Omega}{16\sigma\kappa_o\sqrt{k/\mu}}\right)^{2/13} \tag{B5}$$

$$\approx 9\ \mathrm{K}\left(\frac{\dot{M}}{10^{-6}M_\odot/\mathrm{yr}}\right)^{2/13}\left(\frac{M_*}{1.5M_\odot}\right)^{1/13} Q_o^{2/13}\left(\frac{r}{70\ \mathrm{AU}}\right)^{-3/13} \tag{B6}$$

This temperature profile is shallow, but still colder than the irradiation temperature at large radii where the disk becomes optically thin.